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6. AUTHOR(S) T. Kailath					
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13. ABSTRACT (Maximum 200 words) The researchers conducted investigations in the following: (1) Structured Matrix Factorization, (2) Application to Root Location and Stability Problems, (3) Interpolation Theory, (4) Time-Variant and Displacement Theory. Certain survey papers were written for special occasions and a set of notes for a special short course in Brazil was published by the University of Sao Paulo.					
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FINAL REPORT

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Recursive Analysis of Matrix Scattering Functions

**Thomas Kailath
Information Systems Laboratory
Department of Electrical Engineering
Stanford University
Stanford, CA 94305-4055**

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ARRAY ALGORITHMS FOR INTERPOLATION AND CONTROL PROBLEMS

This is the final report on our work under Contract AFOSR91-0060, for the period November 1, 1990 through October 31, 1993.

The list of publications in Section 2 gives a good indication of the nature and amount of the research carried out during this period. Section 3 lists the personnel supported by the project and also contains abstracts of the two Ph.D. theses completed during the contract period. Section 4 lists various other activities, including conferences attended, special lectures, honors, etc. Section 7 gives an overview of the various topics covered in the research, with more details in the various publications cited below.

1 Research Summary

The papers in the publications list are organized under the following major subheadings: J denotes Journal Papers, C - Conference Papers, A - Papers Accepted for Publication, and S - Papers Submitted for Publication.

1.1 Structured Matrix Factorization

A surprisingly large number of problems in various fields reduces to problems of triangular or orthogonal factorization of matrices. What distinguishes the different applications is the identification of various kinds of structure that allows the matrix factorization to take place in a reasonable amount of time or in a way that allows use of parallel computation or hardware solution via systolic arrays, etc.

We have found that a notion called *displacement structure* encompasses a large variety of structures and applications. The exploration of this theme has continued successfully during this contract period.

Four papers have appeared in print [J2,J5,J7,J9], one has been accepted [A2] and will appear in January 1994, a conference paper [C1] was presented in an invited session at the 1991 Mathematical Theory of Networks and Systems Symposium in Kobe, Japan. Two papers [S6] and [S7] are under review.

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1.2 Applications to Root Location and Stability Problems

Despite a history of more than 100 years, there is a continuing interest in the problem of determining efficient algorithms for determining the number of roots of a given polynomial that lie in a given region, *e.g.* the left half plane, or the unit disc.

A variety of algebraic and analytic tools have been used for this purpose. Our approach is through a theorem of Hermite which reduces the problem to one of determining the inertia of certain so-called Bezoutian matrices. It turns out that these matrices have displacement structure, and so are amenable to our theories. The 16-page journal paper [J1] is quite comprehensive. A review of the major ideas is [J8], appearing in a book dedicated to Professor E.I. Jury, one of the leading workers in this field. A short note on a special test is to appear shortly [A1].

1.3 Interpolation Theory

The problem of determining a function that takes specified values at specified points in the complex plane also has a very long history going back to Lagrange, Euler, Pade and others. Recently these problems have attracted renewed attention because of relations to the partial realization problem of linear system theory, with several papers by Antoulas, Anderson, Ball, Willems and others. Our paper [A4] shows that the theory of displacement structure can give a more efficient computational procedure for solving such problems.

Constrained interpolation problems are those in which the interpolation function is required to be analytic and bounded in certain regions (especially the unit disc) or analytic and with positive real part in the complex plane. Such functions arise in stability theory and circuit synthesis, and more recently have been encountered in the recently much-studied subject of H_∞ -control theory. There are several recent books and monographs espousing various approaches to such problems, *e.g.* Foias and Frazho via Commutant Lifting Theory, Dym via Reproducing Kernel Hilbert Spaces, Ball-Gohberg-Rodman via the pole-zero structure of Rational Matrix Functions. We have found that a solution can be found as a fallout of our fast matrix factorization algorithms exploiting displacement structure. Certain cascade networks can be naturally associated with the recursive factorization algorithm. Such cascades have certain "blocking" properties associated with the zeros of the networks and these properties

can be translated to interpolation properties. The resulting point of view is quite physical and quite general. It is described in the papers C4, S2, and S5.

Specific applications to the so-called four-block problem of H_∞ -theory appear in [C5] and [S1].

1.4 Time-Variant Displacement Theory

Motivated by certain problems in state-space estimation and adaptive filtering (see Sec. 5 below), we generalized the earlier theories to encompass time-variant matrices. The displacement structure theory appears to extend very nicely, and gives new solutions to time-variant interpolation problems recently encountered in various applications, especially in model matching. Our results are in the accepted papers [A3] and [A5], and in the paper [S8] currently under review.

1.5 State-Space Structure: Chandrasekhar Equations and Adaptive Filtering

Perhaps the most studied structure in system theory is state-space structure. It is an interesting question to combine this structure with displacement structure – the result is, for time-invariant systems, just the fast Chandrasekhar equations for Kalman filtering discovered in the early 70's. The new connections led us to extensions of these equations to certain special time-variant systems – see [C3], [S3], [S9]. We discovered that this special structure was just the one that arose in a new state-space formulation of the exponentially weighted least-squares adaptive filtering problem. This is an area of great current interest in signal processing, with several books and monographs and papers in almost every issue of the *IEEE Transactions on Signal Processing*. Our reformulation gives a very simple connection to the Chandrasekhar equations, allowing compact and numerically attractive square-root algorithms – see [C7], [C8], and [S10]. In the current contract period, certain papers based on work in earlier contract periods also appeared – [A3] and [A5].

1.6 General

Certain survey papers were written for special occasions – [A4] for the Kalman 60th birthday volume, and [S4], a long review of our work of the last decade, invited for SIAM Review. A set of notes for a special short course in Brazil was published by the University of Sao Paulo [C2]. We have been pleased to see the increasing interest in our work from numerical analysts and others.

2 AFOSR Supported Publications

Journal and Book Papers

- [J1] H. Lev-Ari, Y. Bistritz, and T. Kailath, Generalized (Bezoutians) and Families of Efficient Root-Location Procedures, *IEEE Trans. Circuits and Systems*, **38**(2):170–186, Feb. 1991.
- [J2] Y. Bistritz, H. Lev-Ari, and T. Kailath, Immittance-Domain Three-Term Schur and Levinson Recursions for Quasi-Toeplitz Complex Hermitian Matrices, *SIAM J. Matrix Anal. and Appl.*, **12**(3):497–520, July 1991.
- [J3] H. Lev-Ari, K-F. Chiang, and T. Kailath, Constrained-Input/Constrained-Output Stability for Adaptive RLS Lattice Filters, *IEEE Trans. Circuits and Systems*, **38**(12):1478–1483, Dec. 1991.
- [J4] T. Kailath, From Kalman Filtering to Innovations, Martingales, Scattering and Other Nice Things, pages 55–88, *Mathematical System Theory: The Influence of R.E. Kalman*, ed. A.C. Antoulas. Springer-Verlag, Communications and Control Engineering edition, 1991.
- [J5] H. Lev-Ari and T. Kailath, State-Space Approach to Factorization of Lossless Transfer Functions and Structured Matrices, *Linear Algebra Appl.*, **162**–**164**:273–295, Feb. 1992.
- [J6] H. Lev-Ari, T. Kailath, and J. Cioffi, Adaptive Recursive-Least-Squares Lattice and Transversal Filters for Continuous Signal Processing, *IEEE Trans. Circuits and Systems*, **39**(2):81–89, Feb. 1992.

- [J7] R. Ackner, H. Lev-Ari, and T. Kailath, Transmission-Line Models for the Modified Schur Algorithm, *IEEE Trans. Circuits and Systems*, **39**(4):290-296, April 1992.
- [J8] T. Kailath, Root Distribution and Displacement Structure, *Symposium on Fundamentals of Discrete-Time Systems*, Chicago, IL, June 1992. (Proceedings published by Albuquerque, NM,
- [J9] A.H. Sayed and H. Lev-Ari and T. Kailath, Fast Triangular Factorization of the Sum of Quasi-Toeplitz and Quasi Hankel Matrices, *Linear Algebra Appl.*, **191**:74-106, Sept. 1993.

Conference Papers

- [C1] T. Kailath and A.H. Sayed, Fast Algorithms for Generalized Displacement Structures, In H. Kimura and S. Kodama, editors, *Recent Advances in Mathematical Theory of Systems, Control, Networks and Signal Processing II, Proc. of the International Symposium on the Mathematical Theory of Networks and Systems*, pages 27-32, Kobe, Japan, June 1991.
- [C2] T. Kailath and A. Sayed, A Short Course on Structured Matrices with Applications in Digital Signal Processing, *9th Brazilian Symposium on Telecommunications*, University of San Paulo, Brazil, Sept. 1991.
- [C3] A. Sayed, H. Lev-Ari, and T. Kailath, Lattice Filter Interpretation of the Chandrasekhar Recursions for Estimation and Spectral Factorization, *Int'l. Conf. on Acoustics, Speech, and Signal Processing*, San Francisco, CA, March 1992.
- [C4] A.H. Sayed and T. Kailath, Recursive Solutions to Time-Invariant Rational Interpolation Problems, *Proc. Int'l. Symp. on Circuits and Systems*, San Diego, CA, May 1992.
- [C5] T. Constantinescu, A. Sayed, and T. Kailath, A Schur-Based Approach to the Four-Block Problem, *American Control Conference*, Chicago, IL, June 1992.
- [C6] T. Kailath, Root Distribution and Displacement Structure, *Symposium on Fundamentals of Discrete-Time Systems*, Chicago, IL, June 1992.

- [C7] A. Sayed and T. Kailath, Structured Matrices and Fast RLS Adaptive Filtering, *2nd IFAC Workshop on Algorithms and Architectures for Real-Time Control*, pp. 211–216, Seoul, Korea, August 1992.
- [C8] B. Khalaj, A. Sayed, and T. Kailath, A Unified Derivation of Square-Root Multichannel Least-Squares Filtering Algorithms, *Int'l. Conf. on Acoustics, Speech, and Signal Processing*, Minneapolis, MN, April 1993.

Papers Accepted for Publication

- [A1] R. Ackner and T. Kailath, On The Ptak-Young Generalization of the Schur-Cohn Theorem, *IEEE Trans. Automatic Control*.
- [A2] R. Ackner, H. Lev-Ari, and T. Kailath, The Schur Algorithm for Matrix-Valued Meromorphic Functions, *SIAM J. Matrix Anal. and Appl.*
- [A3] A. Sayed, T. Constantinescu, and T. Kailath, Square-Root Algorithms for Structured Matrices, Interpolation, and Completion Problems, *IMA Volumes of Mathematics and Its Applications*.
- [A4] T. Boros, A. Sayed, and T. Kailath, Structured Matrices and Unconstrained Rational Interpolation Problems, *Linear Algebra Appl.*,
- [A5] A. Sayed, H. Lev-Ari, and T. Kailath, Time-Variant Displacement Structure and Triangular Arrays, *IEEE Trans. Acoustics, Speech, Signal Processing*.

Papers Submitted for Publication

- [S1] T. Constantinescu, A.H. Sayed, and T. Kailath, A Schur-Based Approach to the Four-Block Problem, *IEEE Trans. Circuits and Systems*.
- [S2] T. Constantinescu, A. Sayed, and T. Kailath, Displacement Structure and Some General Interpolation Problems, *SIAM J. Matrix Anal. and Appl.*
- [S3] T. Kailath, A. Sayed, and H. Lev-Ari, Generalized Chandrasekhar Recursions from the Generalized Schur Algorithm, *IEEE Trans. Automatic Control*.

- [S4] T. Kailath and A. Sayed, Schur Complements and the Interplay between Matrix and Function Theory, *Operator Theory: Advances & Appls.*
- [S5] A. Sayed, T. Kailath, H. Lev-Ari, and T. Constantinescu, Efficient Recursive Solutions of Rational Interpolation Problems, *Integral Equations and Operator Theory.*
- [S6] A. Sayed and T. Kailath, Triangular Factorization of Toeplitz-Like Matrices with Singular Leading Minors, *SIAM J. Matrix Anal. and Appl.*
- [S7] A. Sayed and T. Kailath, Fast Algorithms for Generalized Displacement Structures and Lossless Systems, *Linear Algebra Appl.*
- [S8] A. Sayed, T. Constantinescu, and T. Kailath, Time-Variant Displacement Structure and Interpolation Problems, *IEEE Trans. Automatic Control.*
- [S9] A. Sayed and T. Kailath, Extended Chandrasekhar Recursions, *IEEE Trans. Automatic Control.*
- [S10] A. Sayed and T. Kailath, A State-Space Approach to Adaptive RLS Filtering, *IEEE ASSP Magazine.*

3 AFOSR Supported Personnel

		Summer '91, '92, '93	Academic '91, '92, '93
Principal Investigator	Thomas Kailath	9 months (20%)	
Research Associate	A. Sayed	9 months (80%)	
Postdoctoral Scholars	P. Algoet	2 months (40%)	7 months (40%)
	T. Constantinescu	1 months (78%)	4 month (100%)
Graduate Students	T. Boros	6 months (50%)	19 months (50%)
	R. Ackner	3 months (50%)	8 months (50%)
	B. Hassibi		4 months (50%)
	P. Park		2 months (50%)

3.1 Abstract of Thesis by R. Ackner: Fast Algorithms for Indefinite Matrices and Meromorphic Functions (Dec. 1991)

One of the most famous algorithms for signal processing is the Schur algorithm, which produces a Lower-Diagonal-Upper (*LDU*) factorization of an $n \times n$ structured matrix in $O(n^2)$ operations (in contrast to $O(n^3)$ for a general matrix). At the same time the algorithm can be used to check the analyticity of a certain complex-valued function associated with the given matrix. Most of the works in the literature in this area are concerned with positive definite matrices and their associated analytic functions. In this thesis we investigate a more general class of indefinite matrices and their associated meromorphic functions.

First we describe a scalar modified Schur algorithm that handles functions with poles. Our derivation of the algorithm is new; it is based on triangular factorization via array transformations. We present transmission-line models for the modified Schur algorithm and describe several applications. The first application is to the problem of the root distribution of a polynomial with respect to the unit disc. Three others are to problems from analytic function theory and linear-algebra; matching of Taylor

coefficients, the factorization of certain indefinite Hermitian matrices, and the Schur-Takagi extension problem. We show that these three problems can be solved using the transmission-line models, with physical properties such as causality and energy conservation playing important roles in the solution.

We also present a modified Nevanlinna algorithm and its application to solving interpolation problems. As a new application, we give a proof of the Ptak-Young generalization of the Schur-Cohn theorem.

Finally we extend the modified Schur algorithms to matrix-valued functions that have a finite number of Smith-McMillan poles inside the unit disc. Using a process of "scalarization" we show how to efficiently implement the algorithm using only elementary scalar operations. We also show how to compute the number of poles of a function using our version of the Schur algorithm. In addition we describe a solution to tangential interpolation problems using the algorithm and we give a simple proof of the connection between the number of poles inside the unit disc of each solution and the inertia of a certain Pick matrix.

3.2 Abstract of Thesis by A. Sayed: Displacement Structure in Signal Processing and Mathematics (March 1992)

The notion of displacement structure provides a natural framework for the solution of many problems in signal processing and mathematics. We investigate its application as a useful and powerful unifying tool for exploiting the J -lossless systems, interpolation theory, H^∞ -control, linear least-squares estimation, spectral factorization, time-variant systems, systolic arrays, and adaptive filtering. The tools used throughout this work are not much more than elementary matrix and linear systems theory. The main feature of the derivation is that it requires very little mathematical background from the reader, since it is entirely based on combining a simple Gaussian elimination step with displacement structure.

We give a unifying and simple derivation of a generalized square-root or array algorithm for factoring (non) strongly regular structured matrices, and emphasize how naturally transmission-line structures, triangular arrays, and embedding relations arise in this context. We stress the fact that transmission-line cascades have useful physical characteristics such as causality, energy conservation, and blocking

properties. The first two have been exploited previously. The blocking property, viz., that signals propagating through the cascade at certain frequencies get annihilated, is here exploited to derive efficient recursive solutions to general rational interpolation problems, which arise in many applications. In particular, we use it to provide a new solution of the so-called four-block problem of H^∞ -optimal control design.

The array algorithms arose in linear least-squares estimation theory. We here extend the so-called Chandrasekhar recursions to a class of time-variant models, enabling a new approach to the study of adaptive filtering algorithms.

An important aspect of our work is that the derivation of the generalized array algorithm is exclusively carried out in matrix domain, working with matrix quantities only, and not in function domain. This promptly allows us to extend the matrix-based derivation very smoothly and easily to the time-variant setting, where we introduce time-variant structured matrices, derive the associated array algorithm, and introduce and solve a general time-variant interpolation problem. The point to stress is that the arguments follow the same general lines as in the time-invariant counterpart, except for certain additional technical details. Under a supplementary condition, we further obtain a substantial simplification of the time-variant transmission-line structure and present a triangular array of lattice sections that solves the interpolation problem. We show that each section is composed of a sequence of elementary rotations and a time-variant tapped-delay filter. We also discuss applications to the recursive least-squares problem.

We finally show how insights from the study of structured matrices can be helpful in simplifying and extending the derivation of many well-known adaptive filtering algorithms. We describe a unified square-root-based derivation of the different adaptive schemes, and point out explicit connections with structured matrices and the corresponding array algorithms. The derivation leads to a description of the various schemes in terms of square-root arrays, where one forms a prearray and then uses a convenient rotation in order to obtain the desired postarray. In this process we have encountered rich connections with algorithms that have been long established in linear least-squares estimation theory, such as the Kalman filter, the Chandrasekhar recursions; and the information forms of the Kalman and Chandrasekhar algorithms.

4 Activities and Honors

4.1 Awards & Honors

- Society Award of the IEEE Signal Processing Society, May 1991.
- Senior Vinton Hayes Fellowship, Massachusetts Institute of Technology, Cambridge, MA, Fall 1991.
- Elected to membership in the Third World Academy of Sciences, 1991.
- Degree of *Doctor Honoris Causa* in Engineering, Strathclyde University, Scotland, May 1992.
- Education Award of the IEEE Circuits & Systems Society, May 1993.

4.2 Keynote and Plenary Lectures

- Keynote lecture, Twenty-Fourth Asilomar Conference on Signals, Systems and Computers, Asilomar, CA, November 1990.
- Plenary lecture, Second SIAM Conference on Linear Algebra in Signals, Systems and Control, San Francisco, CA, November 1990.
- The Charles Edison Lecture, University of Notre Dame, Indiana, April 1991.
- Keynote Lecture, Workshop on Fast Algorithms and Parallel Architectures, Bonas, France, July 1991.
- Plenary Lecture, Applied Linear Algebra for Signal Processing Workshop, (IMA Symposium), Minneapolis, MN, April 1992.